The Role of the Core in Irregular Fluctuations of the Earth's Rotation and the Excitation of the Chandler Wobble [and Discussion]

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The role of the core in irregular fluctuations of the Earth's rotation and the excitation of the Chandler wobble

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Transfer of angular momentum between the core and the mantle seems to be the only quantitatively satisfactory explanation of the irregular fluctuations in the length of the day, although some small part, hitherto supposed to be noise in the data, may have an origin in the atmosphere. Correlation between the westward drift, as determined from the motion of the off-centre dipole, and the changes in the length of the day since 1820 seems convincing support for the theory.

The nature of the coupling between the core and the mantle is controversial: if it is electromagnetic, the quantitative difficulty has not been resolved. I have shown that if the torques resulting from electric currents induced in the lower semiconducting mantle are of an impulse character, those parallel to the axis (required to change the length of the day) are of the same order of magnitude as those in the equatorial plane, which may be the long-sought origin of the excitation of the Chandler wobble. Recent observational evidence of the geomagnetic secular variation, and from the Earth's rotation, supports this view: the short timescale changes in the geomagnetic field at the core surface have been greatly underestimated.

1. Irregular changes in the length of the day and the westward drift

One of the most intriguing and difficult problems in discussing the core is its interaction with the mantle. As the lower part of the mantle is a semiconductor there is a possibility, even a likelihood, that the magnetic variations generated in the core, the geomagnetic secular variation, will induce currents in the lower mantle and cause variations in the Earth's rotation as observed by surface observatories. It was Spencer Jones who first pointed out that the only geophysical phenomenon connected with the solid Earth that has a similar timescale to the irregular fluctuations in the length of the day is the geomagnetic secular variation. But it was some time before a simple interpretation of the irregular fluctuations in the rate of rotation of the Earth of about a few milliseconds was suggested. The clue was the rediscovery of a feature of the secular variation: the westward drift of the geomagnetic non-dipole field. Over a century from 1780 to 1885 a pattern reflecting the non-axial part of the field, lines of equal difference between the observed angle of magnetic inclination and that corresponding to a geo centric axial dipole field, was shown by Bauer (1895) to drift to the west. In the course of a century the contours and the zero line moved to the west by about 24° without the pattern's changing. This phenomenon has been rediscovered many times, notably by Vestine *et al.* (1947) and has its simplest interpretation in the idea that the core is rotating at present more slowly than the mantle, i.e. in a westward direction, carrying the lines of magnetic field with it.

The irregular fluctuations in the length of the day since 1650 appeared to be far too large and occurred on too short a timescale to be caused by geological processes affecting the moment of inertia. Moreover, earthquakes, although associated with large displacements of mass, fail quantitatively and searches for correlations between the phenomena were unsuccessful. It followed, from the interpretation of the westward drift of the non-axial parts of the geomagnetic field by rotation of the core relative to the mantle, that changes in the latter would involve changes in the length of the day. On the assumption that the core rotates as a whole during such transfer of angular momentum, the change (increase) in the length of the day, δT (milliseconds) and the change (increase) in the westward drift velocity, $\delta\Omega$, (degrees of longitude per year) were given by Runcorn (1954) as

$$\delta T = -67\,\delta\Omega. \tag{1}$$

Evidence was sought for variations in the rate of westward drift that might be correlated with the irregular fluctuations in the length of the day. Vestine (1953) suggested that the latitude and longitude of the eccentric dipole, used to model the dipole and the quadrupole components of the geomagnetic field, could be determined back to Gauss's first spherical harmonic analysis. He gave plots of the latitude, with which we are not concerned, and the longitude of the position of the eccentric dipole with time and argued that there was some suggestion of a correlation between the westward drift and the irregular fluctuations in the length of the day in the correct sense for conservation of angular momentum. The data used are a very smooth set due to Brouwer. It is quite clear from the long and involved history of the reduction of observations on the longitude of the Moon (De Sitter 1927; Spencer Jones 1939), from which evidence for the irregular fluctuations was found, that the smoothing techniques used probably smoothed out some of the sharpest changes. There is a famous one just before 1900 that Brouwer (1952) has spread over some years but recently it has been shown by van Flandern that this increase in the length of the day of 3 ms probably occurred in a few months (van Flandern 1980).

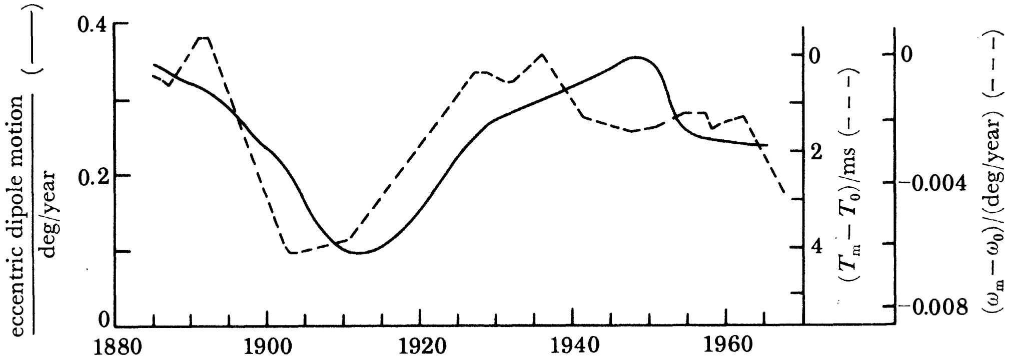

FIGURE 1. Changes in the westward drift velocity and the irregular fluctuations (from Kahle *et al.* 1969).

In recent years a good correlation between the changes in the length of the day and the westward drift velocity, measured by the westward motion of the position of the eccentric dipole, modelling the dipole and quadrupole terms, has been found (see figure 1). The constant of proportionality is, however, not that of (1). From Kahle *et al.* (1969), I calculate

$$\delta T = -16\,\delta\Omega. \tag{2}$$

Thus the component with periods of decades only involves the transfer of angular momentum between the mantle and an outer shell, 200–300 km thick, of the core. Although the question of the electromagnetic torques that explain this coupling is not resolved, it may be intuitively

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reasonable that for the shorter timescales the outer core, but for much longer timescales the whole core, couples to the mantle.

If the rotation rate of the mantle changes as a result of interchange of angular momentum with the atmosphere or because of changes in moment of inertia of the Earth – and Jeffreys (1982) and Etkins & Epstein (1982) suggest polar ice-cap changes – then $\Delta t = -658\,\Delta\Omega$. It will be seen from figure 1 that the observational data rather definitely exclude such explanations of the irregular variations in the length of the day.

2. The non-tidal acceleration in the Earth's rotation

In determining the secular accelerations in the Sun and Moon by the study of ancient eclipses, the existence of a non-tidal term in the secular change in the Earth's rotation has been long suspected. Its existence was first noticed by Urey (1952) in connection with his hypothesis that the core would not have formed at once in an initially cold accreted Earth but might have separated gradually throughout the Earth's life, in which case its moment of inertia would have been diminishing secularly. The idea that the non-tidal term in the Earth's rotation is an acceleration has been taken by Lyttleton (1980) as evidence in favour of his theory (based on an idea of Ramsey (1948)) that the core is silicate transformed by high pressure into a metallic state. He postulates that the core radius is increasing as a result of the dependence on temperature of the transformation of the silicate of the mantle into a metallic state and that in consequence the radius of the Earth is decreasing. The uncertainties in the astronomical data have perhaps played a part in keeping discussion of this muted, but recent studies of the timing of Babylonian lunar eclipses by Stephenson & Morrison (1981) have established the reality of, and determined a value for, this non-tidal term. The increase in the length of the day, mainly the result of the tides, is now given as 1.78 ms per century, and Stephenson & Morrison have shown that the non-tidal part of opposite sign is about one third of this, i.e. 0.8 ms per century. This can be explained by an interchange of momentum between the core and the mantle without the necessity of invoking changes in the radius and moment of inertia of the Earth. This would imply that over the last 25 centuries the day would have shortened by 0.02 s, had there been no effect of tidal friction, and by equation (1) the relative velocity of the core would have changed by 0.30° a year: thus the rotation of the whole core could in the early part of this period have been eastward. In this connection the determination from archaeomagnetic data and the continuous palaeomagnetic record of sufficient resolution to study the secular variation, such as are now available from lake bottom deposits, is important. It was shown (Runcorn 1955) that a westward-drifting field would in general result in a clockwise rotation of the geomagnetic field vector and an eastward drift an anticlockwise rotation. Some evidence of an eastward drift over the time period from 2500 B.P. to 1000 B.P. has been found and appears to support the above argument. Although the model proposed by Bullard *et al.* (1950) to explain the drift has been widely accepted, that the Coriolis force accounts for the outer part of the core rotating more slowly than the inner, another model has been proposed in which the rotation at any one epoch is the cumulative effect of many impulsive torques on the mantle (Runcorn 1954).

3. The core and the excitation of the Chandler wobble

In a short paper at an I.A.U. symposium on the Earth's rotation (Runcorn 1968) I argued that, on the basis of orders of magnitude, the electromagnetic torques exerted by the core on the mantle might be responsible for the excitation of the Chandler wobble. It was pointed out later to me that in a paper by Munk & Hassan (1961), this possibility had been explored previously and they had found that the components of torques on the mantle about an equatorial axis were ineffective in moving the pole by a factor of about 300. Because this factor is about equal to the ellipticity of the Earth, Rochester (1968) supposed that I had misunderstood the role of the Earth's equatorial bulge in stabilizing the Earth's pole of rotation. I therefore wrote another paper (Runcorn 1970) showing that the problem that Munk & Hassan (1961) had solved was the effect on the mantle of a step function torque applied by the core about an equatorial axis, whereas I had supposed the action of the core was that of an impulsive torque. I showed that there was in fact no contradiction between our results and gave a physical explanation of why the factor of about 300 arose and argued that impulsive torques rather than step function torques were a more likely explanation of the observations of the irregular fluctuations in the length of the day.

However, in his recent book, Lambeck (1981) writes:

> Electromagnetic core–mantle coupling provides a quite inadequate means of transferring angular momentum from the core to the mantle in order to sustain the Chandler wobble. Runcorn (1968) argued that the Chandler wobble is maintained by electromagnetic torques but he gives no details and his conclusion is clearly at variance with the results of Rochester & Smylie (1965). More recently Runcorn (1970) proposed that high frequency, localized geomagnetic secular variations exert impulsive torques on the mantle and these would perturb both the length of the day and the wobble. Again, Runcorn gives no details or even order of magnitude estimates of these torques... .

It is thus necessary to clarify this matter once again.

What I did in my short (1968) paper was to argue simply that the torques required to change the length of the day were, according to the astronomical data, short-lasting. Astronomers had of course argued whether the data on the discrepancies between the observed and calculated longitudes of the Moon were to be interpreted in terms of straight lines (corresponding to a number of instantaneous changes in the rate of the Earth's rotation) or, as Brouwer maintained, by a series of quadratic curves (corresponding to sudden changes in otherwise constant torques on the mantle). Might not torques of similar origin excite the Chandler wobble?

From a physical point of view Euler's equation is best expressed in vector form as

$$\boldsymbol{L} = \dot{\boldsymbol{H}} + \boldsymbol{\omega} \times \boldsymbol{H}, \tag{3}$$

where $\boldsymbol{L}$ is the external torque applied to the body, $\boldsymbol{H}$ is its angular momentum measured with reference to coordinates rotating in space with angular velocity $\boldsymbol{\omega}$. Let C be the moment of inertia about the axis of symmetry and A the moment of inertia about a line in the equatorial plane, assuming axial symmetry, then:

$$\boldsymbol{H} = C\omega_3 \boldsymbol{k} + A\omega_2 \boldsymbol{j} + A\omega_1 \boldsymbol{i}.$$

The notation of Munk & MacDonald (1960) is useful: $\omega_3 = \Omega(1+m_3)$, $\omega_2 = m_2\Omega$ and $\omega_1 = m_1\Omega$ and $\boldsymbol{m} = m_1 + \mathrm{i}m_2$, where for small nutation, the squares of m_1, m_2, m_3 and m can be neglected.

Thus $L_3 = C\dot{m}_3\Omega$, the torque required to cause the irregular changes in the length of the day, and

$$L_2 = A\dot{m}_2\Omega - m_1\Omega^2(C-A),$$

$$L_1 = A\dot{m}_1\Omega + m_2\Omega^2(C-A),$$

or

$$\dot{\boldsymbol{m}} + (1/\mathrm{i})\,\boldsymbol{m}\sigma_{\mathrm{r}} = \boldsymbol{L}_{\mathrm{w}}/A\Omega, \tag{4}$$

where $\boldsymbol{L}_{\mathrm{w}} = L_1 + \mathrm{i}L_2$, the torque required for maintaining the wobble, and $\sigma_{\mathrm{r}} = \Omega^2(C-A)/A$.

Solving (4) by Laplace transform method, the subsidiary equation is

$$\{p + (\sigma_{\mathrm{r}}/i)\}\,\overline{\boldsymbol{m}} - \boldsymbol{m}_0 = \overline{\boldsymbol{L}}_{\mathrm{w}}/A\Omega \tag{5}$$

$$\therefore \boldsymbol{m} = (\boldsymbol{m}_0 + \boldsymbol{H}_{\mathrm{w}}/A\Omega)\exp(\mathrm{i}\sigma_{\mathrm{r}}t), \tag{6}$$

where H_{w} is the impulsive torque at $t = 0$ discussed above, and m_0 is the pole position at $t = 0$.

This result is that given by Runcorn (1968), i.e. the application of the impulsive torque H_{w} in the equatorial plane displaces the pole of rotation on the Earth's surface in the direction of vector H_{w} by an amount $H_{\mathrm{w}}/A\Omega$ and thereafter the pole moves over the surface in a circle, but the centre of this remains the pole of figure as before, neglecting the damping. It is incorrect to state, as it sometimes has been stated, that the simplified discussion in Runcorn (1968) is erroneous: the impulsive torques required to excite the Chandler wobble are within an order of magnitude of those inferred to exist from the irregular changes in the length of the day.

However, a different conclusion is reached if it is supposed that a unit function torque in the equatorial plane is applied to the mantle: the physics of the situation is then different.

If

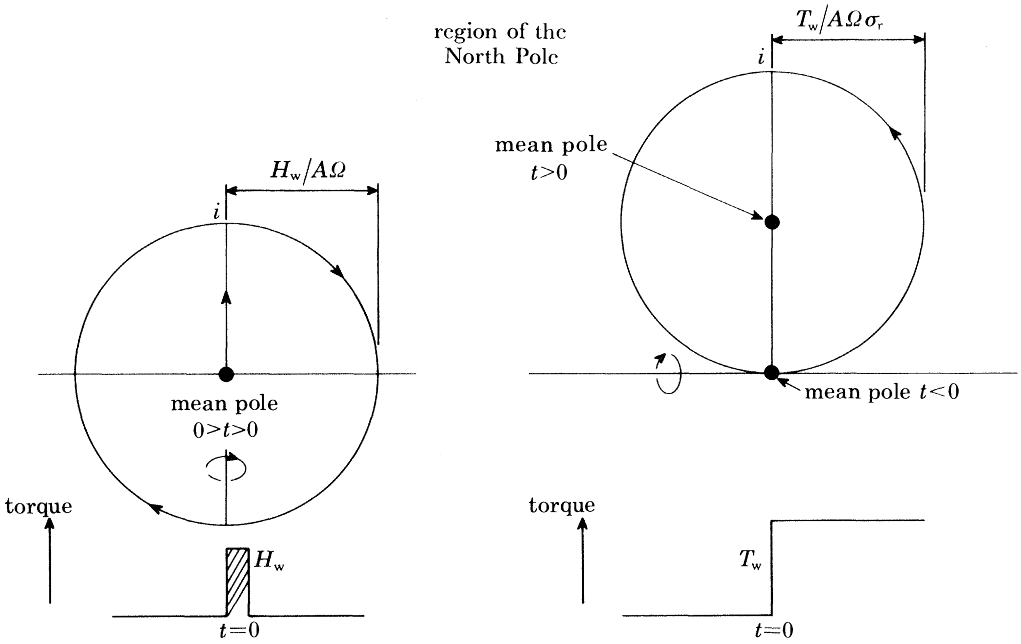
$$\begin{aligned} \boldsymbol{L}_{\mathrm{w}} &= 0 \quad \text{for} \quad t < 0 \\ &= 1 \quad \text{for} \quad t > 0, \end{aligned}$$

equation (5) gives $\boldsymbol{m} = \mathrm{i}L_{\mathrm{w}}\{1 - \exp(\mathrm{i}\sigma_{\mathrm{r}}t)\}/A\sigma_{\mathrm{r}}\Omega$ for $t > 0$. Then $L_{\mathrm{w}}/L_3 = \sigma_{\mathrm{r}}|m|/\dot{m}_3$.

Taking the amplitude of the Chandler wobble as 0.14″ and $\dot{m}_3$ as 8×10^{-9} per year, $L_{\mathrm{w}}/L_3 \approx 400$, essentially the result of Munk & Hassan (1961).

The physics of the two contrasting cases are shown in figure 2. It should be noted that the one on the right also fits the case of excitation by an earthquake, the hypothesis being that a sudden change of the axis of figure is produced.

Because a factor of about 400 came up, Rochester supposed that the ellipticity of the Earth was involved and that my argument had failed to treat properly the question of gyroscopic motion. But the difference in the physics of these two cases, i.e. where an impulsive torque is applied about an equatorial axis and where a step function torque is applied, is quite subtle. It is perhaps easiest to think of the Earth rotating initially about its axis of figure, this axis, the axis of angular momentum and the axis of instantaneous rotation being coincident initially. If an impulsive torque is applied the axis of instantaneous rotation, or rather the axis of angular momentum, because the two are very close together, is moved to a new point and then Chandler motion commences. If, however, a step function torque is applied the axis of angular momentum and the axis of instantaneous rotation, which are close together, move away from the axis of figure in a circular path with its centre about a new pole, which is not the axis of maximum moment of inertia. If earthquakes were responsible for the excitation of the Chandler wobble, this pole would be the new axis of maximum moment of inertia (the effect of mass displacements is to alter the axis of figure). Smylie & Mansinha (1968), who first proposed that earthquakes were responsible for the excitation of the Chandler wobble, searched for correlations between earthquake occurrences and the times when the polar path abruptly changed its shape. The reason for the famous factor of 400 is that in the model on the right of figure 2, the torque is permanently displacing the mean centre of the motion of the Earth's rotation from the axis of figure and then, of course, the centrifugal forces acting upon the equatorial bulge produces a large restoring torque against which the applied torque works. The mean motion is about the axis of figure in the model on the left but not in that on the right. So although it has been held repeatedly that the basic principles of the theory of solid body mechanics prevent the core from being a plausible source of the excitation of the Chandler wobble, its interaction with the mantle is, in principle, capable of explaining both the changes in the length of the day and the excitation of the wobble.



impulsive torque about equatorial axis

step function torque about equatorial axis or sudden displacement at $t=0$ of pole of figure

FIGURE 2. Physical principles of the excitation of the Chandler wobble.

I suggested (Runcorn 1968) that, as the core exerts from time to time impulsive torques parallel to the axis of rotation on the mantle capable of changing the length of the day by 3 ms (*ca.* 1900 and 1920), it is reasonable to suppose that the core also from time to time exerts impulsive torques along an equatorial axis, thus 'causing the excitation of the Chandlerian nutation'. To effect this transfer of angular momentum between the core and the mantle, we must postulate the existence of an impulsive torque equal to $4\times10^{-8}\,H$, where H is the angular momentum of the Earth's mantle. Although the amplitude of the motion of the pole lies between $0.1''$ and $0.2''$, the jumps in the pole path that might be taken to be associated with its excitation are nearly an order of magnitude less: a typical jump is $0.04''$. As a sudden displacement of the pole of rotation through $0.04''$ requires an impulsive torque equal to $2\times10^{-7}\,H$, my conclusion remains that as the effects are within an order of magnitude and as the geomagnetic secular variation is strongly non-axial the core could be the cause of the excitation of the Chandler wobble.

This argument suggests a test: the recent studies of the Chandler wobble. The only recent case in which a kick has been observed has been studied by Guinot (1972*a*). He shows that it fits the model on the left better than that on the right of figure 2. In figure 3 are fortnightly positions of the pole with the annual term taken out. Before about March–April 1967 the path was a circle of radius 0.1″; after that time the radius was 0.05″. Guinot calculated the

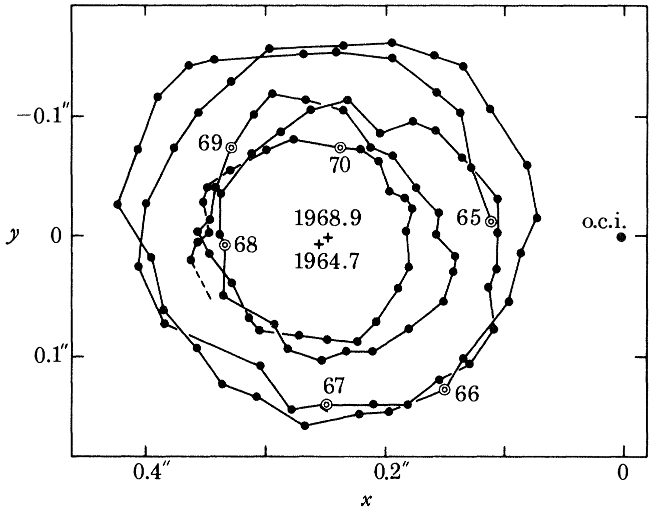


FIGURE 3. The Chandler wobble (polar motion with annual term removed) (from Guinot 1972*a*).

centre of the circular polar motion before and after the kick, and figure 3 shows that they are close together. This is what would have occurred if an impulsive torque was the cause of the kick. Guinot (1972*b*) discusses other cases where the radius of the Chandler motion changes but not its centre. In conclusion I think that we have underestimated the shortness of the timescales of the interaction between the core and mantle.

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Discussion

K. Lambeck (*Laboratoire de Géophysique et Géodynamique Interne, Université Paris-Sud, France*). The excitation function corresponding to the electromagnetic core–mantle coupling model may be written as

$$\psi = \Delta\psi H(t)\,\mathrm{e}^{-at},$$

where $H(t)$ is the Heaviside function and a is the decay constant. The motion $m(t)$ of the rotation axis, relative to Earth-fixed axes, is of the form (Lambeck 1980; p. 59)

$$m(t) = \mathrm{j}\sigma_0/(a+\mathrm{j}\sigma_0)\;\Delta\psi(\mathrm{e}^{-at}-\mathrm{e}^{-\mathrm{j}\sigma_0 t}),$$

where $\mathrm{j} = (-1)^{\frac{1}{2}}$ and σ_0 is the Chandler frequency. At time $t = 0$ the excitation pole jumps to a new position, e.g. from $\psi = 0$ to $\psi = \mathrm{j}\sigma_0/(a+\mathrm{j}\sigma_0)$, and then drifts back to its original position with a time constant a^{-1}. This model differs from that of Runcorn, who considers a delta function excitation which would seem to imply a time constant a^{-1} that is very short compared with the period of the Chandler wobble. If the formalism of Rochester (1968) and Rochester & Smylie (1965) is used, the requisite lower-mantle conductivity would have to approach that of the core (Lambeck 1980, p. 265).

It should also be noted that Sasao *et al.* (1977) point out that if such delta-like torque operate, the nearly diurnal nutations of the Earth would be excited very efficiently.

My second point is that some insight into the core–mantle coupling required to explain the decade changes in length of day may be obtained through an examination of the time series of the length of day in the spatial domain. Are changes in length of day gradual, taking place over several years, or are they abrupt, over a few months? It has not been possible to examine this previously because zonal wind contributions mask these effects. We have therefore attempted to 'correct' the astronomical data for the atmospheric contribution (Lambeck & Hopgood 1982). The 'corrected' time series points to a rather substantial change in the proportional change in length of day in 1972, although it is premature to say that the latter event marks a substantial change comparable with that observed around 1900–05. It is perhaps tempting to associate it with the change in magnetic field parameters discussed by Le Mouël *et al.* (1981).

The low-frequency part of the spectrum of the length of day may throw some light on the nature of coupling. If the applied torques occur over short intervals of time, less than a year, say, then the spectrum of the first derivative of the change in length of day will be relatively devoid of power at low frequencies, whereas if the changes occur over long intervals the power at low frequencies will be enhanced. The data suggest the former (Lambeck 1980). There may therefore be an argument, albeit tenuous, that changes in the magnetic field at the core–mantle boundary may take place with a very short time constant.

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S. K. **Runcorn**. Sasao *et al.* (1977) do indeed state: 'It has been suggested by Runcorn (1970) that the Chandler wobble and the irregular changes in the length of day may both be excited by common impulsive torques at the core–mantle boundary. If that is the case, our result obtained in equation (67) predicts also the appearance of the free core nutation as large as the Chandler wobble!' From their equation (67) which they discuss they reach the same conclusion of the relative effectiveness of the step function and the impulse as I did in my 1970 paper, although they do not state that they do. An impulsive torque represented by a Dirac delta function is of course a mathematical fiction – for the statement by Sasao *et al.* to be true the torque on the mantle would have to be applied over a time of less than a day: I have made no such suggestion.

For a current system in the lower mantle to decay in less than a year and thus, by interaction with the main field, to cause the impulsive torque that I suggest could excite the Chandler wobble, the electrical conductivity of the inner part of the mantle must be that of a semiconductor and two or three orders of magnitude less than the core (Rochester 1968; Rochester & Smylie 1965). However, I did suggest that, as magnetohydrodynamic disturbances are transmitted with the Alfvén velocity, which in the core is of the order of 1 cm s^{-1}, and as the dimensions of the eddies in the core inferred from the anomalies in the geomagnetic field are of the order of some hundreds of kilometres, the torques with a timescale similar to that inferred from the data on the Earth's rotation, 1 month to 1 year, are not impossible. As the electrical conductivity arising from semiconduction in the lower mantle is that which gives a free decay time of current distributions in the mantle of the same order, this is satisfactory.

Regarding the many calculations that have been done concerning the electromagnetic torques on the mantle, they have suffered from a lack of knowledge of the fields actually present at the surface of the core. The discovery of the discontinuities in the geomagnetic secular variation rates may be of great significance in this connection.

D. E. **Winch** (*Department of Applied Mathematics, University of Sydney, Australia*). From the 'rolling cone' analysis of the free motion of a rigid body, it is clear that the Chandler wobble or free Eulerian nutation arises as a consequence of the non-coincidence of the axis of angular momentum and the axis of figure, and that the Chandler wobble, once started, will be perturbed by any variation in angular momentum that changes the orientation of the angular momentum axis. Would it not be more appropriate to speak of perturbation of the Chandler wobble by such small changes in angular momentum rather than excitation, which seems to imply an *ab initio* generation of the wobble?

S. K. Runcorn. Without excitation, the Chandler wobble would disappear, reducing its amplitude to 1/e in about 15 years (H. Jeffreys, *The Earth*, 6th edn, Cambridge University Press (1976)). Perturbations or disturbances in the polar path can therefore be reasonably associated with the excitation mechanism. The point at issue is whether the mechanism is in the Earth's core or in earthquakes.